J. Fluid Mech. (1065), *vol.* 23, *part* 3, *pp.* 583–584 583
Printed in Great Britain

Note on Murray's paper on bubbles in fluidized beds

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(Received 26 April 1965)

Murray's equation (53) for the velocity of rise of a three-dimensional (spherical) bubble of diameter d_B can be written

$$U_B = [gd_B/6c]^{\frac{1}{2}}, \tag{1}$$

where the unassigned coefficient c enters Murray's analysis in the linearization of the convective momentum term in the momentum equation for the solids (17). (Equations (53) and (17) are as numbered in Murray 1965*b*.) This compares with the expression

$$U_B = K[\tfrac{1}{2}gd_B]^{\frac{1}{2}} \tag{2}$$

used by Rowe & Partridge (1965), who determined the velocity coefficient, K, for several fluidized systems by measuring bubble diameter and velocity from

			Min. fluidization velocity	$F = 2U^2_{\mathrm{m.f.}}/gd_B$ Range of observed bubble sizes	
Particle diam. d_p (μ)	Veloc. coeff. $K = U_B/[\frac{1}{2}gd_B]^{\frac{1}{2}}$	Murray's coeff. $c = 1/3K^2$	$U_{\mathrm{m.f.}}$ (cm/sec) interstitial	minimum $d_B = 1\frac{1}{2}$ cm	maximum $d_B = 5$ cm
		Ballotini (glass spheres)			
550	0·835	0·478	61·6	5·16	1·55
460	0·853	0·458	43·7	2·60	0·779
220	0·921	0·393	10·3	0·144	0·0433
170	0·930	0·385	5·62	0·0429	0·0129
140	1·000	0·333	3·55	0·0171	0·00514
120	0·899	0·412	2·41	0·00790	0·00237
82	1·010	0·327	1·09	0·00162	0·000485
60	0·912	0·401	0·67	0·000611	0·000183
		Silver sand			
500	0·871	0·440	45·2	2·78	0·835
460	0·890	0·421	34·9	1·66	0·497
330	0·912	0·401	21·1	0·610	0·183
230	0·998	0·335	12·7	0·219	0·0659
140	1·050	0·302	5·01	0·0341	0·0102
82	1·090	0·281	2·10	0·00599	0·00181
72	1·210	0·228	1·50	0·00306	0·000919
		Acrylic granules			
121	0·916	0·397	1·48	0·00298	0·000893
		Magnesite			
240	0·935	0·381	9·00	0·110	0·0330
		Crushed coal			
410	0·961	0·361	22·8	0·706	0·212

TABLE 1. The data of Rowe & Partridge

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X-ray ciné photographs. Values of c calculated from the measured values of K are listed in table 1, from which it is seen that the coefficients are not truly constant. The observed values of Murray's c are also appreciably less than $c = 1$ or $c = \frac{3}{5}$, the two values tentatively suggested by mathematical reasoning, and Murray's figure 13, based on earlier velocity measurements over a limited range of particle sizes, is not a critical test of the value or the constancy of c.

Murray defines a Froude number, $F = 2U^2_{\mathrm{m.f.}}/gd_B$, where $U_{\mathrm{m.f.}}$ is the minimum (interstitial) fluidization velocity and he suggested that c may indeed be a function of F and the observed values are plotted to discover this in figure 1. The coefficient c is plotted to a linearly scaled ordinate as a horizontal line spanning the logarithmically scaled range of F over which K was originally measured. There is no obvious and simple relationship.

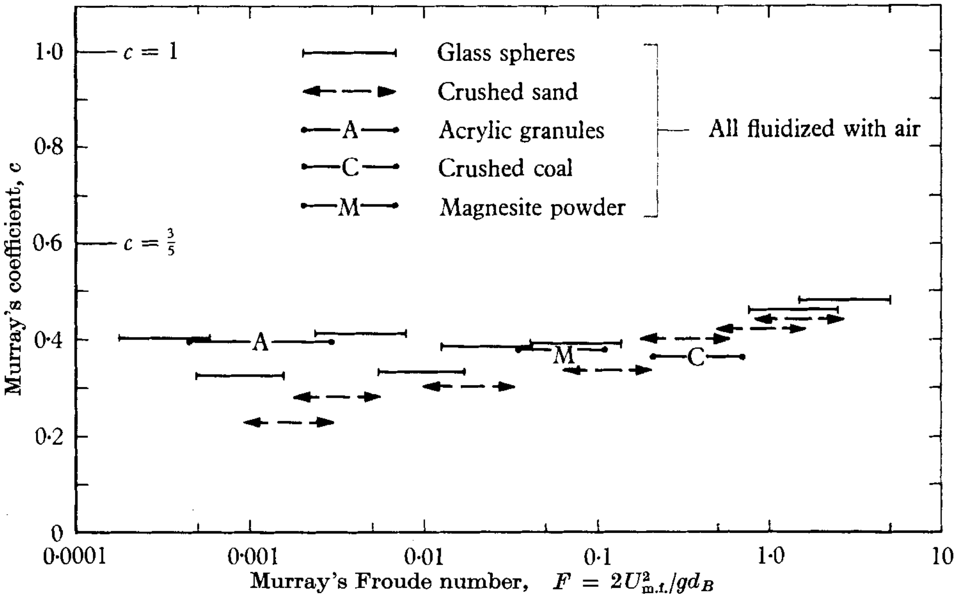

FIGURE 1. Observed values of c plotted against F.

Murray's c is analogous to a drag coefficient and, considered in this way, it might be expected to be a function of Reynolds number describing solids flow around the rising bubble. It is not clear, however, what 'viscosity' should be assigned to a fluidized system in order to calculate a Reynolds number but at least this 'viscosity' might be expected to vary inversely as the particle spacing. Murray (1965*a*) postulates a shear viscosity which varies inversely as the particle spacing and a bulk viscosity which varies inversely as the cube of particle spacing. The velocity coefficient, K, has in fact been shown to increase with the porosity of the particulate system at the point of minimum fluidization as is seen in figure 15 of Rowe & Partridge (1965). This corresponds to a reduction in drag coefficient with particle spacing.

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